MAT 1033C
Final Exam Review
Solutions (Kincade)

(1) $(-2)^4 = 16$

(2) $\frac{3a^0 - b^0}{3b^0} = \frac{3(1)-1}{3(1)} = \frac{3-1}{3} = \boxed{\frac{2}{3}}$

(3) $\left(\frac{2xy^4}{x^{-3}y}\right)^{-2} = \frac{2^{-2}x^{-2}y^{-8}}{x^6y^{-2}} = \frac{y^2}{2^2x^6x^2y^8}$

$= \boxed{\frac{1}{4x^8y^6}}$

(4) $-2(x-3) = 3(2x-6)$
$-2x+6 = 6x-18$
$6+18 = 6x+2x$
$24 = 8x$
$\boxed{3 = x}$

(5) $\frac{2x}{3} - \frac{x}{4} = 5$

lcd 12

$4(2x) - 3(x) = 5(12)$
$8x - 3x = 60$
$5x = 60$
$\boxed{x = 12}$

(6) $P(x) = -x^{\textcircled{4}} + 3x - 1$ degree = 4

(7) $Q(x) = -2x^4 - 3x + 17$
$Q(0) = -2(0)^4 - 3(0) + 17 = \boxed{17}$

(8) $(6y^3 - 4y^2 + y) - (-4y^2 + 3y - 1) + (y^3 + 4)$
$6y^3 - 4y^2 + y + 4y^2 - 3y + 1 + y^3 + 4$
$\boxed{7y^3 - 2y + 5}$

(9) $-3abc^3(4abc - c^2)$
$\boxed{-12a^2b^2c^4 + 3abc^5}$

(10) $(x-4y)^2$
$(x-4y)(x-4y)$
$x^2 - 4xy - 4xy + 16y^2$
$\boxed{x^2 - 8xy + 16y^2}$

(11) $(3x+7)(2x-5)$
$6x^2 - 15x + 14x - 35$
$\boxed{6x^2 - x - 35}$

(12) $x+2 \overline{)4x^3+8x^2-6x}$ $\quad 4x^2-6+\frac{12}{x+2}$

$-4x^3 \mp 8x^2$

$\pm 6x \pm 12$

12

(13) $4a^3b^2+16a^4b-4a^3b^3$

$4a^3b(16b+4a-b^2)$

(14) $2y^2-98$

$2(y^2-49)$

$2(y+7)(y-7)$

(15) $16a^3+54b^3$ Recall $a^3+b^3=(a+b)(a^2-ab+b$

$2(8a^3+27b^3) = 2(2a+3b)(4a^2-6ab+9b^2)$

"a" $=2a$ "b" $=3b$

(16) $xy^2-4x+3y^2-12$

$x(y^2-4)+3(y^2-4)$

$(x+3)(y^2-4)$

$(x+3)(y-2)(y+2)$

(17) $x^2-3x-18$

$(x-6)(x+3)$

(18) $6x^2-21x-12$

$3(2x^2-7x-4)$

$3(2x+1)(x-4)$

(19) $6x^2-3x=0$

$3x(2x-1)=0$

$3x=0 \quad 2x-1=0$

$x=0 \quad 2x=1 \quad x=\frac{1}{2}$

(20) $x(x-5)=6$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x-6=0 \quad x+1=0$

$x=6 \quad x=-1$

(21) $\frac{-64x^4y^8}{24x^3y^{10}}$

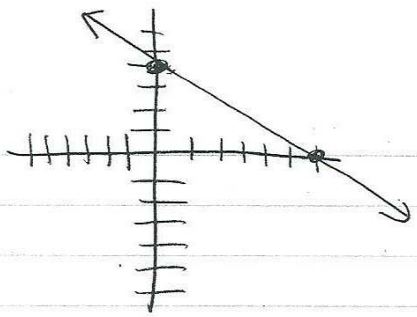
$= \frac{-8x}{3y^2}$

(22) $\frac{\frac{6x}{5y}}{\frac{x}{10}} =$

$\frac{6x}{5y} \cdot \frac{10^2}{x} = \frac{12}{y}$

(23) $2x + 3y = 12$

x	y
0	4
6	0



(24) (1, 1)
(5, 3)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{1-3}{1-5} = \frac{-2}{-4} = \frac{1}{2}$

(25) $7x + 5y = 35$ (solve for y)

$5y = -7x + 35$

$y = \frac{-7}{5}x + \frac{35}{5}$

$y = \frac{-7}{5}x + 7$

$y = mx + b$

$m = -\frac{7}{5}$

(0, 7)

(26) $4 \text{ \& } \frac{-1}{4}$ are (1) since they are opposite reciprocal slopes

$m_1 m_2 = -1$

(check: $4\left(\frac{-1}{4}\right) = -1$)

(27) $m = -2$
(6, 8)

$m = \frac{y - y_1}{x - x_1}$

$\frac{-2}{1} = \frac{y-8}{x-6}$

$-2(x-6) = 1(y-8)$

$-2x + 12 = y - 8$

$-2x + 20 = y$

$y = -2x + 20$

(28) (4, 0)
(0, 8)

$m = \frac{y - y_1}{x - x_1}$

$m = \frac{0-8}{4-0} = \frac{-8}{4} = -2$

$-2 = \frac{y-0}{x-4}$

$-2(x-4) = y$

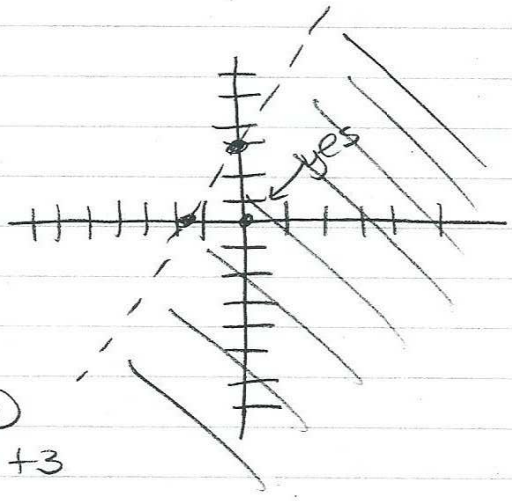
$-2x + 8 = y$

$y = -2x + 8$

(29) $(3,4)$ $(-2,4)$ same y-coordinate so $\boxed{y=4}$

(30) $y < 2x+3$

x	y
0	3
$-3/2$	0



dotted

test (0,0)

$0 < 2(0)+3$

$0 < 3$

true

(31) $\left(\frac{27}{125}\right)^{-2/3} = \left(\frac{125}{27}\right)^{2/3} = \left(\frac{5}{3}\right)^2 = \boxed{\frac{25}{9}}$

2 → square it

3 → cube root (first)

(32) $(81x^4y^8)^{3/4} = 81^{3/4}x^3y^6 = \boxed{27x^3y^6}$

$81^{3/4} = 3^3 = 27$ (cube 4th root)

(33) $6^{2/3} \cdot 6^{4/3} = 6^{6/3} = 6^2 = \boxed{36}$

(34) $\sqrt{32m^9n^7}$

$= \boxed{4m^4n^3\sqrt{2mn}}$

2	32
2	16
2	8
2	4
	2

groups of 2

(35) $\sqrt[3]{-2000x^5y^7}$

$(x\,x\,x)\,x\,x$

$(y\,y\,y)(y\,y\,y)\,y$

$-10xy^2\sqrt[3]{2x^2y}$

2	2000
2	1000
2	500
2	250
5	125
5	25
	5

groups of 3

(36) $\sqrt{\frac{108xy^6}{3xy^2}} = \sqrt{36y^4} = \boxed{6y^2}$

(37) $\frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \boxed{\frac{3\sqrt{7}}{7}}$

need 1 more 7 for a group of 2

(38) $\frac{\sqrt[3]{10}}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \boxed{\frac{\sqrt[3]{20}}{2}}$

	have	need
$4 =$	$2 \cdot 2$	2

(39) $\sqrt{x^2+8x+16} = \sqrt{(x+4)^2} = \boxed{x+4}$

(40) $\sqrt[3]{24} - 2\sqrt[3]{81} = 2\sqrt[3]{3} - 2 \cdot 3\sqrt[3]{3}$

$= 2\sqrt[3]{3} - 6\sqrt[3]{3}$

$= \boxed{-4\sqrt[3]{3}}$

2	24
2	12
2	6
	3

3	81
3	27
3	9
	3

(41) $\left(-\frac{3}{4}\sqrt{3a}\right)\left(8\sqrt{12a}\right)$

$= -\frac{24}{4}\sqrt{36a^2} = -6(6a) = \boxed{-36a}$

(42) $\frac{3}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{3\sqrt{2}+3}{\sqrt{4}+\sqrt{2}-\sqrt{2}-1} = \frac{3\sqrt{2}+3}{1}$

conjugate

$= \boxed{3\sqrt{2}+3}$

(43) $\sqrt[3]{2-x} - 3 = 0$

$\sqrt[3]{2-x} = 3$

cube both sides

$(\sqrt[3]{2-x})^3 = 3^3$

$2 - x = 27$

$-x = 25$

$\boxed{x = -25}$

(44) $\sqrt{x^2+5} = 5+x$

square both sides

$(\sqrt{x^2+5})^2 = (5+x)^2$

$x^2+5 = (5+x)(5+x)$

$x^2+5 = 25+5x+5x+x^2$

$5 = 25+10x$

$-20 = 10x$

$\boxed{-2 = x}$

check

$\sqrt{(-2)^2+5} =$

$5-2$

$\sqrt{9} = 3$

$3 = 3$

ok

(45) $x(2x-11) = -12$

$2x^2-11x+12 = 0$

$(2x-3)(x-4) = 0$

$2x-3=0$

$2x=3$

$\boxed{x=\frac{3}{2}}$

$x-4=0$

$\boxed{x=4}$

(46) $(x-2)^2 - 3 = 0$

$(x-2)^2 = 3$

$x-2 = \pm\sqrt{3}$

$\boxed{x = 2 \pm \sqrt{3}}$

(47) $x^2-6x+11=0$

$x^2-6x+\frac{9}{(\frac{-6}{2})^2} = -11+9$

$(x-3)^2 = -2$

$x-3 = \pm\sqrt{-2}$

$\boxed{x = 3 \pm i\sqrt{2}}$

(48) $\underset{a}{4}x^2 \underset{b}{-2}x \underset{c}{-3} = 0$

① $D = b^2 - 4ac$
$= (-2)^2 - 4(4)(-3) =$
$= 4 + 48$
$= 52$

② $\sqrt{D} = \sqrt{52} = 2\sqrt{13}$

$2 \mid 52$
$2 \mid 26$
13

$x = \frac{-b \pm \sqrt{D}}{2a}$

$x = \frac{-(-2) \pm 2\sqrt{13}}{2(4)}$

$x = \frac{2 \pm 2\sqrt{13}}{8}$

$x = \frac{2}{8} \pm \frac{2\sqrt{13}}{8}$

$x = \frac{1}{4} \pm \frac{\sqrt{13}}{4}$

or $\frac{1 \pm \sqrt{13}}{4}$

(49) $z^2 + 64 = 0$
$z^2 = -64$
$z = \pm\sqrt{-64} = \pm 8i$

(50) $(2 + \sqrt{-4})(2 - \sqrt{-1}) = (2 + 2i)(2 - i)$
$= 4 - 2i + 4i - 2i^2$
$= 4 + 2i - 2(-1) = 6 + 2i$
$+2$

(51) $|7-4| + |-5| - (-4)$

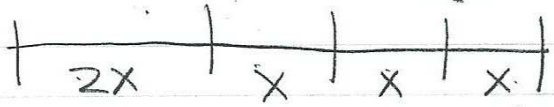
$|3| + |-5| + 4 = 3 + 5 + 4 = 12$

(52) $x = 2$
$y = -3$
$z = -1$

$\frac{xy + 6z}{y(x - 2z)} = \frac{(2)(-3) + 6(-1)}{(-3)(2 - 2(-1))}$

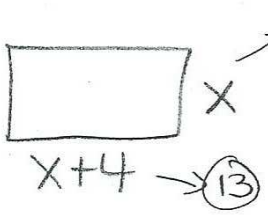
$= \frac{-6 - 6}{-3(2+2)} = \frac{-12}{-12} = 1$

(53)



$5x = 200$
$x = 40$

longest piece $= 2x = 2(40)$
$= 80$ ft

(54) 

$P = 44m$

Area $= (9)(13)$ (8)
$= \boxed{117m^2}$

$(x+4)+(x)+(x+4)+(x) = 44$
$4x+8 = 44$
$4x = 36$
$x = 9$

(55) $\boxed{|5x-3| = -4}$

no soln since $|\ |$ = pos #

Recall usual set up is $5x-3=-4$ & $5x-3=4$
then ✓ your answers

(56) $\frac{x^2+2x-3}{x^2-9} \cdot \frac{x^2-2x-3}{x-1} = \frac{(x+3)(x-1)}{(x+3)(x-3)} \cdot \frac{(x-3)(x+1)}{(x-1)}$

$= \boxed{x+1}$

(57) $\frac{x^2-9}{x^3+8} \cdot \frac{x^2-2x+4}{x-3} =$ Recall cube formula

$= \frac{(x+3)(x-3)}{(x+2)(x^2-2x+4)} \cdot \frac{(x^2-2x+4)}{(x-3)} = \boxed{\frac{x+3}{x+2}}$

cube formulas: $a^3+b^3 = (a+b)(a^2-ab+b^2)$
$a^3-b^3 = (a-b)(a^2+ab+b^2)$

(58) $\frac{4x+2}{x-3} - \frac{2x+8}{x-3} = \frac{4x+2-2x-8}{x-3} = \frac{2x-6}{x-3}$
(has lcd)

$= \frac{2(x-3)}{(x-3)}$

$= \boxed{2}$

(59) $\frac{3x-1}{x-4} + \frac{x-1}{4-x} = \frac{3x-1}{x-4} - \frac{x-1}{x-4}$

$= \frac{3x-1-x+1}{x-4} = \boxed{\frac{2x}{x-4}}$

(60) $\frac{x+2}{2x-4} - \frac{2}{x-2} = \frac{x+2-2(2)}{\text{lcd}}$

$2(x-2)$

lcd $= 2(x-2)$

$= \frac{x+2-4}{\text{lcd}}$

$= \frac{x-2}{2(x-2)} = \boxed{\frac{1}{2}}$

(61) $\frac{\frac{2}{1} - \frac{1}{y}}{\frac{y}{1} - \frac{1}{2}} = \frac{2(2y)-1(2)}{y(2y)-1(y)} = \frac{4y-2}{2y^2-y}$

lcd $2y$ (multiply by it)

$= \frac{2(2y-1)}{y(2y-1)} = \boxed{\frac{2}{y}}$

(62) $\frac{1}{a} - \frac{1}{4} = \frac{-1}{2a}$

lcd $4a$

$4a\left(\frac{1}{a}\right) - 4a\left(\frac{1}{4}\right) = 4a\left(\frac{-1}{2a}\right)$

$4(1) - a(1) = 2(-1)$

$4 - a = -2$

$-a = -6$

$\boxed{a = 6}$

(63) $\frac{1}{x+3} = \frac{3}{x-1}$

lcd $(x+3)(x-1)$

$x \neq -3, 1$

$1(x-1) = 3(x+3)$

$x - 1 = 3x + 9$

$-10 = 2x$

$\boxed{-5 = x}$

(64) $1 + \frac{x+2}{x+3} = \frac{5}{3}$

lcd $3(x+3)$

$x \neq -3$

$1(3)(x+3) + (x+2)(3) = 5(x+3)$

$3x + 9 + 3x + 6 = 5x + 15$

$6x + 15 = 5x + 15$

$6x - 5x = 15 - 15$

$\boxed{x = 0}$

(65) $\frac{3x}{x-2} - 4 = \frac{14-4x}{x-2}$

lcd $x-2$, $x \neq 2$

$3x - 4(x-2) = 14 - 4x$

$3x - 4x + 8 = 14 - 4x$

$3x = 14 - 8 = 6$

$x = 2$

but $x \neq 2$ so no soln

(66)

hose 1	6 hrs
hose 2	x
together	4 hrs

lcd $12x$

$\frac{1}{6} + \frac{1}{x} = \frac{1}{4}$

$2x + 12 = 3x$

$12 = x$

hose 2 will take 12 hrs by itself

(67)

Paula	3 hrs
Bill	2 hrs
together	x

lcd $6x$

$\frac{1}{3} + \frac{1}{2} = \frac{1}{x}$

$2x + 3x = 6$

$5x = 6$

$x = 6/5$

1⅕ hrs together

(68) $y = \frac{4}{x^2}$

(10)

denom $\neq 0$

so $x^2 \neq 0$

$x \neq 0$

$D = \mathbb{R}$ except 0

(69) $f(x) = x^2 + 4x + 7$

AKA $f(x) = (x+2)^2 + 3$

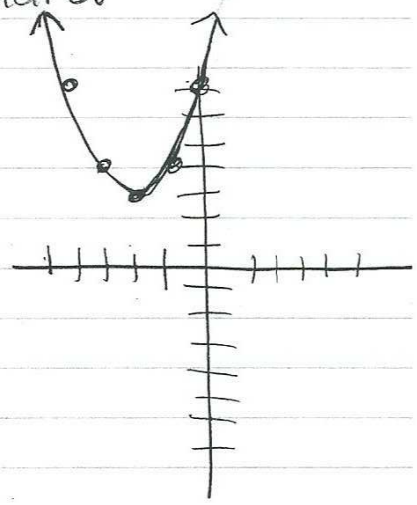
parabola

vertex $= (-2, 3)$

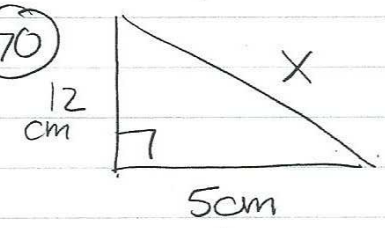
up

standard

x	y
−4	7
−3	4
−2	3
−1	4
0	7



(70)



$a^2 + b^2 = c^2$

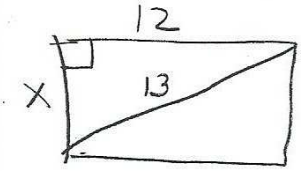
legs, hypotenuse

$12^2 + 5^2 = x^2$

$144 + 25 = x^2$

$169 = x^2$

$x = 13$ cm

(71) 

$x^2 + 12^2 = 13^2$

$x^2 + 144 = 169$

$x^2 = 25$

$\boxed{x = 5}$

(72) $(2-3i) - (3+4i) = 2 - 3i - 3 - 4i$

$= \boxed{-1 - 7i}$

(73) $\frac{1+i}{i} \cdot \frac{i}{i} = \frac{i + i^2}{i^2} = \frac{i-1}{-1} = -i + 1$

$= \boxed{1 - i}$

(74) $\frac{3 + \sqrt{-1}}{2 - \sqrt{-4}} = \frac{3+i}{2-2i} = \frac{3+i}{2(1-i)}$

$\frac{(3+i)}{2(1-i)} \cdot \frac{(1+i)}{(1+i)} = \frac{3 + 3i + i + i^2}{2(1 + i - i - i^2)} = \frac{2+4i}{2(2)}$

$i^2 = -1$, $-(-1)$

↑ conjugate

$= \frac{2+4i}{4} = \boxed{\frac{1}{2} + i}$

(75) $A = P + PRT$ for P

$A = P(1 + RT)$

$\boxed{\frac{A}{1+RT} = P}$

(76) yes — passes the vertical line test

(77) ① $4x - 14y = -15$

② $18x - 12y = 9$

LCM $(x) = 36$

9 ① $36x - 126y = -135$

-2 ② $-36x + 24y = -18$

$-102y = -153$

$y = \frac{153}{-102} = \frac{51 \cdot 3}{-51 \cdot 2} = \frac{-3}{2}$

$4x - 14\left(\frac{3}{2}\right) = -15$

$4x - 21 = -15$

$4x = +6$

$x = 6/4 = 3/2$

$\boxed{\left(\frac{3}{2}, \frac{3}{2}\right)}$

(78) $\frac{5x}{x+7} \cdot \frac{x^2+8x+7}{10}$

$= \frac{5x}{x+7} \cdot \frac{(x+7)(x+1)}{10} = \boxed{\frac{x(x+1)}{2}}$

Recall $|x|<3$ is an intersection (12)

(79) Natural

$\{2, \frac{9}{3}, \sqrt{4}\}$

(80) rational

$\{2, 0, -3, \frac{1}{2}, \frac{9}{3}, \sqrt{4}\}$

(81) $|3-2x|=5$

$3-2x=5$ & $3-2x=-5$

$-2x=2$ $-2x=-8$

$\boxed{x=-1}$ $\boxed{x=4}$

both are ok

✓ $\boxed{x=-1}$ $|3-2(-1)|=5$

$3+2=5$

$5=5$

✓ $\boxed{x=4}$ $|3-2(4)|=5$

$|-5|=5$

$5=5$

(82) $2(x+3) \le 4$

$2x+6-4 \le 0$

$2x+2 \le 0$

$2x \le -2$

$\boxed{x \le -1}$

$(-\infty, -1]$

(83) $2 < \frac{x+4}{2} < 4$ lcd 2

$-4 > x+4 > -8$

$-8 < x+4 < -4$

$-4 \quad -4 \quad -4$

$\boxed{-12 < x < -8}$

$-12 \quad -8 \quad (-12, -8)$

(84) $|x+1| \ge 2$

$x+1 \ge 2$ or $x+1 \le -2$

$x \ge 1$ $x \le -3$

(union)

$(-\infty, -3] \cup [1, \infty)$

$(-\infty, -12) \cup (6, \infty)$ →

(85) $\left|\frac{x}{3}+1\right| - 4 > -1$

$\left|\frac{x}{3}+1\right| > 3$

$\frac{x}{3}+1 > 3$ (or) $\frac{x}{3}+1 < -3$

$x+3 > 9$ $x+3 < -9$

$x > 6$ (or) $x < -12$